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FINAL Report

Project 346

REPORT

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Investigation of: Apsidal Motion of Stars

Subject of Report: Final Report

Submitted by: G. Keller and D. N. Limber

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THE PHOTOELECTRIC LIGHT CURVES OF YY SAGITTARII AND RS CANUM VENATICORUM.

Purpose

As stated in the contract schedule, the purpose of this project was to conduct a photoelectric and spectroscopic study of binary stars having measurable apsidal motion. In the course of the work it was found to be more efficient to concentrate on the photoelectric studies, and this was done.

Introduction

The theory of apsidal motion[1,2,3,] has shown that there is a close relationship between the degree of central mass condensation of the components of a binary star system and the rate of advance of the line of apsides. Compilations of the best existing observational data[4,5,6,] have shown that for most stars these condensations are similar to those of polytropes whose indices lie between 3 and 4. There may also be a trend toward higher condensations for later spectral types, although in view of the uncertainties in the data, this statement cannot be made with assurance.

The theory of stellar interiors involves many unknown parameters which describe the relative abundances of the elements at different points within a star. In addition, the theory is subject to serious uncertainties as to which physical processes govern the dynamical condition of the stellar material at various depths within the star.

1. Russell, M. N. 88, 641, 1928.
2. Cowling, M. N. 98, 734, 1938.
3. Sterne, M. N. 99, 451, 1938.
4. Russell, Ap. J. 90, 641, 1939.
5. Sterne, M.N. 99, 662. 1939.
6. Keller, Ap. J. 108, 347, 1948.

Consequently, it is very desirable to imporve the observational data on the apsidal motion of stars as much as possible in order to provide another reliable criterion with which to judge the merits of the various proposed stellar models. In addition to the periods of the apsidal motion, it is essential that the relative radii of the components of the binary be determined accurately, since the computations depend sensitively on these quantities. If the orbital eccentricity is large, it too must be known with some precision. A reasonable estimate of the mass ratio also is needed. For these reasons, as well as the need for high precision in determining the displacement of secondary minimum, photoelectric observations are indicated.

Two stars, YY Sagittarii and RS Canum Venaticorum have been well observed, and a third, RU Monocerotis, is currently under observation. In this report we shall give the light curves of the first two stars, and certain preliminary observations with regard to their apsidal motion. Mr. Nelson Limber, an arts graduate student at Ohio State University, is now analysing these curves in order to obtain their orbital elements.

The Instrument

All observations have been made with the 36 inch reflector at the Steward Observatory, Tucson, Arizona. The photometer, whose sensitive element is a 1P21 photomultiplier tube, was designed by Carpenter and Wood. The A. C. amplifier was designed and built by Gartlein. Recording is performed on a Leeds and Northrup Speedomax.

It was found that the amplification is not strictly linear, the deflection varying as about the 0.9 power of the signal. In view of this fact the instrument was frequently calibrated by means of artificial

light sources. The internal consistancy of various calibrations made under differing experimental conditions lead us to believe that the calibration is good to about 0.5 per cent. The source of the non-linearity is apparently in the amplifier, and not in the optical system.

Observational Procedure

During all of the observations at least two comparison stars were used, and in many cases three. In this way a running check on the calibration was obtained, assuming, of course, that the comparison stars themselves were not variable. Alternate two minute runs on the variable stars and comparison stars were employed. Shorter runs were not feasible because of short period fluctuations in the background. The mean sky intensity was measured after every few stellar observations.

Reductions

All observations were first corrected by the calibration curve, and then for the sky background. Differential extinctions were corrected for on the basis of the $\Delta m = 0.35 \sec \zeta$ law. Several independent determinations of the coefficient 0.35 gave the same result to $\pm$ 0.02, using both type A2 and F5 stars. Since no filter was used with the photometer, there is reason to be concerned about the change in extinction coefficient with wave length.

Several special runs were made to establish the relative luminosities of the comparison stars. Thereafter, when computing its luminosity, the variable was always compared with the average of the preceding and following comparison star observations, regardless of which ones these stars happened to be.

<u>YY Sagittarii</u> (B.D. - 19° 5148; $18^h\ 41^m\!.7$; - 19° 27' (1950) AO.)

This star was discovered to be a variable by Miss Cannon[7]. It was first observed visually by Zinner[8] who, obtained a period of 1.31 days. Kordylewski[9] published the first light curve. He found that Zinner's period should be doubled, and moreover that the secondary minumum is markedly displaced, a result which Zinner[10] subsequently confirmed. Shapley and Keller[11] and later Shapley and Miss Swope[12] published results based on measurements of the Harvard patrol plates. They suggested the following ephemeris for primary minimum, which has been employed throughout by the present authors:

Pr. Min. 2419467.0871 + 2.6284841 E. (1).

Shapley and Swope found that the depths of primary and secondary minima were 0.55 and 0.53 magnitudes respectively. They estimated the period of apsidal motion to be greater than 300 years, and the eccentricity to be greater than 0.15. They detected no difference in the widths of minima. Russell[13], partly on the basis of the near equality of the widths of minima, estimated that e = 0.17 and P' = 350 yrs. Later Sterne[5] computed the period of apsidal motion on the basis of some additional unpublished measures of Miss Swope, employing the elements $r_1/A = 0.127$ and $r_2/A = 0.127$ due to Mrs. Shapley. Sterne's results, based on a least square fitting to the data, are e = 0.140 ± 0.010 and P' = 282 ± 49 yrs.

7. Pickering, H. C. 137,1908.
8. Zinner, A. N. 190, 377, 1912; 195, 460, 1913, Astr. Ab. 40, 1922.
9. Kordylewski, A. A. c. 1, 95, 1930.
10. Zinner, A. N. 239, 60, 1930.
11. H. Shapley and K.W. Keller, H. B. 893, 6, 1933.
12. H. Shapley, and Swope, H. B. 909, 9, 1938.
13. Russell, Ap. J. 90, 647, 1939.

In the present study the following comparison stars were used:

B. D.	Harvard Pg. Magnitude	Spectrum	Relative P.E. Luminosity
D -19°5140	9.55	A0	1.201 ± .007
E -20°5243	9.80	F5	1.000
F -19°5147	10.09	A2	0.657 ± .005

Individual observations in the neighborhoods of the minima are given in Table I, and are plotted in Figures 1 and 2. Luminosities are referred to comparison star E as 1.000. The probable error per observation is about 1.3%. Normal points lying between the minima are given in Table II. Each point combines 10 observations. The entire light curve will be found in Figure 3.

The phases of minima at various epochs, referred to the ephemeris (1), are given in Table III. Dr. Shapley and Miss Swope have very kindly permitted the authors to present their results in greater detail than heretofore. Since $e \cos \omega$ is approximately equal to $\pi/2 \left[P_s - P_p - \frac{1}{2}\right]$ for small e, where P_s and P_p are the phases of secondary and primary minimum respectively for any epoch and ω is the longitude of periastron, it is possible to compute $e \cos \omega$ as a function of epoch. A plot of these data is to be found in Figure 4. Each point on the figure is surrounded by a box indicating the extent of the probable error. The point at 1893.5 is very uncertain depending on only two observations of subnormal brightness near secondary minimum. It is not known whether these observations lie on the ascending or descending side of the minimum. The top and bottom of the shaded box correspond to these two possibilities.

It is clear that attempting to fit a cosine curve to Figure 4 is at best an uncertain process, and it is not impossible that the period of

apsidal motion is very long. The eccentricity (which would equal the amplitude of such a curve) is certainly greater than 0.15; with the existing data, Russell's estimate of e=0.17 and P' =350 years seems fairly reasonable.

RS Canum Venaticorum: (B.D. + 36°2344; $13^h08^m.5$; +36°12' (1950); F4n, G8)

The variability of this star was discovered by Mme. Caraski[14] in 1914. The orbital period was first determined by Hoffmeister[15] to be approximately 4.797 days. In a later paper[16] he suggests that the period may be variable, and refutes a suggesting by Maggini[17] that the period should be only 2.4 days. Baker and Cummings published a correction to Hoffmeister's period, and found that there is a period of constant light during primary minimum. The first analysis of the light curve (based on the 4.8 day period) was published by Schneller[19]. His ephemeris,

$$\text{Pr. Min. } 2423579.344 + 4.797944\ E, \qquad (2)$$

is the one which has been used in reducing the present observations. Schneller's observations suggest the existence of a secondary minimum, and the observations made between eclipses indicate flat maxima of equal height.

Sitterly[20] has published two light curves, one based on the Harvard patrol plates, and the other based on observations with the Princeton visual photometer.

14. Ceraski, A.N. 197, 256, 1914.
15. Hoffmeister, A.N. 200, 178, 1914.
16. Hoffmeister, A.N. 208,249, 1918.
17. Maggini, Arcetri Pub. 34, 64, 1916.
18. Baker and Cummings, Laws Bull. 2, 150, 1916.
19. Schneller, A.N. 233, 361, 1928.
20. Sitterly, Princeton Contr. 11, 21, 1930.

The analysis of the observed times of minima shows a definite non-linear relation between the phases of observed primary minima (with reference to his own ephemeris) and the epoch of observation. He suggests that there may be a 26 year sinusoidal variation in the orbital period. Sitterly also observed that the primary minimum of his visual light curve was asymmetrical, particularly outside of eclipse. In addition, the maximum following primary minimum averaged about 0.07 magnitude brighter than the other maximum. He attempted to explain this phenomenon by assuming the simultaneous existence of a tidal lag and of differences in brightness between the advancing and following sides of the brighter star. He was unable to obtain a reasonable fit with the observed data on this basis, and so left the cause of the asymmetry unaccounted for. On the other hand, Sitterly's photographic light curve does not show the asymmetry, and has constant amplitude outside primary minimum. The difference between the two light curves is partly due to the lower accuracy of the photographic measurements.

The Harvard plates were also measured by Miss Pilsworth. A light curve based on these new measurements is given by Mrs. Payne-Gaposchkin[21]. The times of primary minimum were also redetermined, thereby strengthening the evidence in favor of a variation in the orbital period. The photographic light curve derived by these workers does exhibit a secondary minimum of about the same depth as that of Sitterly's visual curve.

There is some evidence that during the epoch of Sitterly's observations the photographic light curve may have shown an inequality in the heights of maxima, and in the same direction Sitterly found visually.

21. C. Payne-Gaposchkin. Proc. Am. Phil. Soc. 81, 189, 1939; Harvard Rpt. No. 170.

Mrs. Payne-Gaposchkin suggests that there may even be a fluctuation in the heights of maxima which is correlated with the fluctuation in orbital period. As she points out, however, the accuracy of her data is such that the support which they give to this suggestion is only of "possible significance".

A spectroscopic study of the system has been made be Joy[22]. He finds that the radial velocity curve does not show a measurable eccentricity of orbit. The spectral type of the brighter, smaller star is F4n, and that of the larger, fainter star is dG8. Joy remarks on the fact, also noted by other observers, that although the absolute magnitude and spectrum would place the fainter component among the dwarfs, its size and mass would place it among the subgiants.

In the present study the following comparison stars[19] were used:

	B. D.	Schneller's Pg. Magnitude	Spectrum	Relative P.E. Luminosity
1.	+35° 2421	7.96	F5	1.560±.020
5.	+35° 2422	8.81	G0	1.000
6.	+35° 2418	9.22	G0	0.556±.010

Individual observations in the neighborhoods of the minima are given in Table IV, and are plotted in Figures 5 and 6. Luminosities are referred to comparison star 5. There appears to be a systematic difference between the luminosities near the bottom of primary minimum as measured on different nights. While it is possible that there are intrinsic fluctuations in the brightness of the fainter star, there is also the possibility that systematic observational errors may be responsible[23].

22. Joy, Ap. J. 72, 41, 1930.
23. We are indebted to Dr. John Irwin for a discussion on this point.

The nights on which higher luminosities were recorded occurred in the late spring, at which time the atmosphere over Tucson is considerably more humid. This would have the effect of obscuring the ultraviolet more strongly than the longer wave lengths, i.e., of making a red star appear brighter when compared to a blue star. The light during primary minimum comes largely from the G8 star, and our comparison stars in this light range are of type G0. The effect of the water vapor absorption would thus be to increase the relative luminosity of the variable during the minimum. The effect would also be increased by the absence of a filter to reduce the amount of ultraviolet radiation transmitted to the photocell. A further study of the data may yield a method of estimating the variable water vapor absorption effect, in which case suitable corrections will be made.

Normal points lying between the minima are given in Table V. Each point combines 10 observations. The entire light curve appears in Figure 7. It will be noted that the maximum which follows primary minimum is unmistakably higher than the other maximum. This confirms both Sitterly's and Mrs. Payne-Gaposchkin's observations for the epoch near J.D. 2423000.

It has occurred to the writers that a clue to the difference in the heights of maxima might be found in similarities between the system RS CVn and U Cephei. The brighter component of U Cep is a rapidly rotating B8 or B9 star[24], which may be compared with the F4n component of RS CVn. The fainter components are G2 and G8 stars respectively. They have lines of normal width and their radii are considerably larger than those of the earlier type companions. The similarity of the light curves of the two

24. O. Struve, Ap. J. 99, 222, 1944.

systems has frequently been noted[25], particularly the asymmetry of the primary minimum near the shoulders. It is interesting to speculate on what would happen if RS CVn were attended by streams of gas in a fashion similar to that proposed by Struve for U Cephei and SX Cassiopeiae. Let it be supposed that a hot stream of gas is flowing away from the leading edge of the F4 star, and a cooler stream is returning to that star's trailing edge from the G8 star. Then the uncovering of the hotter stream following primary eclipse would tend to give the sharp rise in the light which follows primary minimum. The same stream would be eclipsed more slowly prior to primary eclipse, since at that time it presumably would be oriented at an appreciable angle to the line of sight. As an extension of the hypothesis, one could argue that after secondary minimum the stream of cooler gas returning to the trailing edge of the brighter star would partially obscure the hotter stream. The effect would be to reduce the total amount of light received during the oncoming period of maximum light. Unfortunately, in the case of U Cephei, the maximum following the secondary minimum appears to be the brighter, so that it is necessary to find some reason why the effect should be reversed.

While it is not felt that the explanation of the asymmetry of primary minimum on the basis of the gas streams can be considered very seriously without supporting spectroscopic evidence, it is worthy of consideration before attempting a precise quantitative analysis of the light curve on the basis of other hypotheses.

25. Sitterly was led to apply Dugan's method of analysis for U Cephei (Princeton Contr. 5, 1920) to RS CVn because of this similarity. A more recent light curve for U Cep is given by Walter, A.N. 276,225, 1948.

The observed phases of primary minimum are assembled in Table VI. Most of these are taken directly from Sitterly's paper after conversion to Schneller's ephemeris (2). Points due to Himpel[26] and the authors have been added. The same material, plus a group of points from Mrs. Payne-Gaposchkin's paper[21] have been plotted in Figure 8. Since the latter author does not give her initial epoch, but implies that her curve agrees substantially with Sitterly's, we have taken the liberty of assuming an epoch so that both curves agree in all but the first few points. The resulting plot suggests that Sitterly's surmise that a cyclic fluctuation of period exists has been substantiated. The length of the cycle may be closer to 35 years or more, however. The amplitude of fluctuation appears to be about 0.009 of an orbital period.

It is not possible to obtain the phase of the secondary minimum with very much precision. By considering only the bottom portion of that minimum the following estimate was made:

Phase of Secondary Minimum near J.D. 2433017: 0.489 ± 0.002.

The displacement of secondary minimum from the position midway between primary minima is therefore

$$1.489 - 0.983 - 0.500 = 0.006 \pm 0.002 \text{ periods.}$$

One may ask whether this is sufficient displacement to allow the cyclic changes in primary period to be explained on the basis of the apsidal motion hypothesis. Unfortunately, neither the time of secondary, nor the present displacement in phase of the primary from zero phase of the mean ephemeris is sufficiently well known. It does appear from Figure 8 that the primary minimum would now be arriving before the prediction of a mean ephemeris, so that in any event the computed displacement of secondary is in the direction which would be required by the apsidal motion theory.

26. Himpel. A.N 261, 233, 1936.

Summary

The photoelectric light curves of the eclipsing binaries YY Sagittarii and RS Canum Venaticorum are presented. It is found that the orbital eccentricity of YY Sgr is greater than 0.15, and may be considerably larger. The primary minimum of RS CVn is asymmetrical near the shoulders, and the maxima are of unequal height. It is suggested that streams of gas may be present in this system and contribute to the asymmetry of the minima. The length of the period of primary minimum appears to vary in a 35 year cycle. The secondary minimum may at present be slightly displaced in the direction to be expected if the variation in primary period is the result of apsidal motion. It is the intention of the writers to analyse the light curves with the object of deriving the elements of the photometric orbits.

Acknowledgments

The writers would like to express their indebtedness to the following individuals who have given valuable assistance in the prosecution of this program: Dr. Edwin Carpenter and Dr. F. Bradshaw Wood, of the Steward Observatory, for permission to work at their observatory, and for their valuable technical advice and assistance; Dr. Newton Pierce for supplying a list of references concerning the stars investigated.

Staff

This work was carried out by the following members of the project staff.

J. Allen Hynek, Supervisor

Geoffrey Keller, Chief Investigator

D. Nelson Limber, Observer

Donald Arveson, Angus Gillis and Paul Orth, Observing Assistants

Miss Mary Bock and Mrs. Constance Slottebak, Computers.

NOTE: In submitting this report it is understood that all provisions of the contract between The Foundation and the Cooperator and pertaining to publicity of subject matter will be rigidly observed.

Investigator [illegible] Date [illegible]

[illegible] Date 4-21-50

Supervisor [illegible] Date [illegible]

FOR THE OHIO STATE UNIVERSITY RESEARCH FOUNDATION

Executive Director [illegible] Date 4/25/50

Table I.

YY SAGITTARII

Primary Minimum

Epoch-Phase	Luminosity	Epoch-Phase	Luminosity	Epoch-Phase	Luminosity
5170.96116	.891	5181.98545	.691	5185.03018	.747
.96522	.887	.98709	.696	.03298	.781
.96546	.891	.98901	.653	.03443	.775
.96731	.890	.99102	.621	.03644	.779
.96935	.866	.99364	.621	.03890	.811
.97088	.854	.99779	.564	.04056	.823
.97334	.836			.04265	.854
.97516	.828	5182.00122	.511	.04468	.847
.97722	.800	.00354	.499	.04624	.860
.97902	.772	.00571	.474	.04825	.876
.97976	.724	.00804	.459	.05026	.888
.98224	.760	.01023	.483	.05197	.882
.98735	.728	.01205	.486	.05401	.884
.98663	.694	.01409	.516		
.98879	.669	.01673	.546		
.99022	.655	.01871	.567		
.99223	.634	.02053	.587		
.99360	.608	.02257	.612		
.99558	.598				
		5184.97757	.762		
5177.01304	.492	.98061	.731		
.01574	.561	.98191	.743		
.03190	.788	.98418	.711		
.03397	.764	.98558	.678		
.04773	.885	.98806	.661		
.05000	.911	.99004	.634		
.05241	.897	.99159	.619		
.05489	.884	.99388	.593		
		.99541	.578		
5180.04354	.805	.99734	.533		
.04618	.806	.99948	.531		
.04872	.830				
.05085	.869	5185.00093	.502		
.05315	.875	.00296	.473		
.05482	.904	.00436	.479		
		.00582	.479		
5181.96056	.892	.00756	.455		
.96304	.896	.00901	.489		
.96569	.889	.01041	.489		
.96833	.836	.01213	.493		
.97089	.844	.01406	.496		
.97332	.839	.01546	.545		
.97506	.798	.01744	.568		
.97723	.803	.01945	.574		
.97945	.786	.02085	.585		
.98114	.776	.02251	.627		
.98296	.761	.02788	.675		

Secondary Minimum

Epoch-Phase	Luminosity
5166.42171	.600
.42325	.642
.42840	.656
.42977	.688
.43167	.728
5169.42186	.571
.42852	.653
.43058	.666
.43710	.723
.44054	.761
.44363	.794
.44682	.834
.44875	.825
.45081	.854
.45221	.877
.45462	.877
.45808	.886
.46006	.893
.46302	.894
.46511	.902
.46690	.911
.46907	.908
5171.36056	.933
.36222	.927
.36436	.906
.36592	.892
.36816	.883
.36999	.862
.37173	.875
.37382	.851
.37561	.840

Epoch-Phase	Luminosity
5177.38564	.691
.39182	.706
.39438	.624
.39665	.607
.39885	.594
.40112	.554
.40374	.551
.40680	.499
.40913	.534
.41140	.528
5180.42064	.571
.42326	.568
.42566	.613
.42867	.655
.43612	.735
5183.46477	.822
.46686	.892
.46937	.917
5185.36224	.909
.36398	.898
.37624	.805
.37880	.787
.38187	.761
.39476	.644
.39838	.594
.39999	.605
.40118	.618
.40324	.560
.41280	.497
.41473	.512

Table II.

YY SAGITTARII

Normal Points

Phase	Luminosity	Probable Error
0.05941	0.895	0.008
.06981	.896	.002
.08935	.884	.003
.11259	.888	.004
.13365	.887	.002
.15326	.900	.003
.16711	.889	.006
.19733	.898	.002
.22241	.908	.003
.25563	.900	.001
.28105	.906	.008
.32599	.917	.005
.35061	.922	.003
.47900	.893	.004
.52232	.909	.004
.58099	.895	.003
.60569	.919	.008
.62449	.920	.002
.64555	.926	.003
.66666	.922	.003
.70333	.898	.003
.72090	.904	.005
.73278	.908	.006
.74523	.911	.004
.77490	.913	.004
.79095	.904	.004
.80649	.897	.003
.82315	.891	.003
.84440	.911	.010
.86047	.896	.004
.88556	.899	.003
.94755	.903	.004

Table III.

YY SAGITTARII

Epoch and Phases of Observed Minima

Epoch		Shapley and Swope Primary		Secondary		e cos ω	
1893.5 ±	3	-0.004 ±	0.005	0.479 ±	0.015	-0.027 ±	0.025
1902.0	2	+ .002	.003	.471	.003	- .049	.007
1908.5	1.5	+ .004	.002	.459	.004	- .071	.007
1913.0	1.3	- .001	.003	.453	.004	- .072	.008
1917.5	1.5	.000	.003	.446	.003	- .085	.007
1922.5	1.5	+ .002	.005	.443	.005	- .093	.011
1927.5	1.5	+ .004	.002	.433	.002	- .112	.004
1932.0	1.3	+ .005	.002	.429	.002	- .119	.004
1936.0	1.3	+ .003	.004	.419	.003	- .132	.008
Kordylewski							
1927.5 ±	1.5	+0.010 ±	0.003			-0.123 ±	0.010
Zinner							
1913.0 ±	1.5	+0.005 ±	0.002	0.449 ±	0.002	-0.088 ±	0.005
Keller and Limber							
1949.44 ±	0.03	0.0075 ±	0.0005	0.4110 ±	0.0005	-0.1516 ±	0.0011

TABLE IV.

RS Canum Venaticorum

Primary Minimum

Epoch-Phase	Luminosity	Epoch-Phase	Luminosity	Epoch-Phase	Luminosity
1965.92008	1.497	1965.97051	.392	1966.99589	.405
.92309	1.526	.97113	.399	.99673	.402
.92598	1.486:	.97191	.388	.99746	.403
.92978	1.431	.97247	.400	.99824	.402
.93082	1.457	.97323	.392	.99889	.429
.93156	1.406	.97381	.409	.99976	.403
.93386	1.397	.97464	.413	1967.00044	.409
.93444	1.358	.97517	.413	.00115	.414
.93507	1.324	.97589	.402	.00310	.452
.93552	1.316	.97644	.396	.00388	.465
.93600	1.313	.97726	.397	.00461	.473
.93668	1.338:	.97786	.407	.00547	.494
.95233	0.728			.00607	.515
.95314	.718	1966.96004	.522	.00669	.533
.95425	.667	.96095	.504	.00759	.551
.95490	.646	.96165	.494	.00824	.577
.95554	.621	.96236	.474	.00935	.610
.95632	.601	.96323	.451	.01022	.628
.95684	.585	.96386	.437	.01138	.670
.95807	.552:	.96460	.410:	.01209	.699
.95862	.545	.96538	.424	.01300	.732
.95929	.519	.96606	.413	.01370	.761
.95981	.511	.96683	.407	.01458	.790
.96052	.516	.96751	.404	.01526	.818
.96111	.476	.96842	.404	.01595	.853
.96182	.452	.96910	.400	.01682	.875
.96234	.452	.97059	.398	.01773	.919
.96304	.429	.97142	.404	.01862	.933
.96354	.424	.97292	.394	.01967	.986
.96433	.423	.97495	.401	.02050	1.003:
.96489	.407	.98177	.396		
.96554	.399	.98298	.390	1969.92126	1.463
.96602	.389	.98567	.420	.93052	1.443
.96667	.394	.98653	.400	.93407	1.389
.96719	.395	.98844	.410	.93488	1.368
.96786	.397	.98971	.385	.93576	1.345
.96838	.397	.99368	.396		
.96910	.391	.99518	.408	1970.92083	1.431
.96972	.389				

TABLE IV. (continued)

RS Canum Venaticorum

Primary Minimum

Epoch-Phase	Luminosity	Epoch-Phase	Luminosity	Epoch-Phase	Luminosity
1970.92212	1.466	1972.99515	0.397	1975.93533	1.386
.92426	1.453	.99595	.403	.93651	1.274
.92677	1.484	.99689	.392	.93730	1.256
.92766	1.464	.99769	.408	.93853	1.302
.92900	1.434	.99870	.414	.93963	1.248
.92986	1.437	.99941	.410	.94041	1.191
.93079	1.420	1973.00073	.404	.94117	1.174
.93215	1.466	.00163	.436	.94228	1.153
.93321	1.411	.00272	.436	.94351	1.079
.93526	1.359	.00346	.466	.94416	1.030
.93644	1.332	.00449	.492		
.93741	1.301	.00808	.578	1976.96161	0.516
.93853	1.274	.00883	.610:	.96261	.482
.93932	1.[illegible]	.01011	.605:	.96371	.469
.94011	1.165	.01090	.635	.96449	.468
.94102	1.177	.01190	.625	.96715	.430
.94204	1.172	.01275	.681	.96812	.433
.94280	1.149	.01536	.806	.96892	.416
.94416	1.079	.01749	.886	.96998	.438
.94503	1.034	.01870	.911	.97109	.417
.94568	1.008	.01985	.991	.97300	.428
.94635	1.006	.02140	1.053	.97423	.433
.94719	0.969	.02216	1.144	.97595	.426
.94788	.940	.02335	1.190	.97695	.43[illegible]
.94878	.904	.02413	1.183	.98038	.436
.94952	.889	.02523	1.213	.98148	.424
.95033	.848	.02603	1.231	.98245	.431
.95117	.808	.02713	1.279	.98392	.428
.95186	.786	.02795	1.288	.98475	.447
.95338	.732	.02986	1.375	.98775	.438
.95408	.718	.03075	1.495	.98911	.419
.95499	.682	.03195	1.459	.99024	.469
.95571	.670	.03428	1.490	.99541	.435
.95667	.629	.03504	1.524	.99641	.444
.95741	.615	.03694	1.553		
.95825	.577	.03804	1.596	1978.00446	.497
.95987	.534	.03888	1.508	.00528	.534
.96081	.526	.04062	1.638	.00653	.576
.96163	.486	.04146	1.534	.00728	.540
.96252	.474	.04263	1.552	.00841	.626
.96324	.469	.04480	1.541	.00916	.633
.96418	.424	.04575	1.590	.01028	.627
.96518	.424	1974.04055	1.618	.01109	.653
.96596	.441:	.04233	1.545	.01226	.718
.96786	.415	.04418	1.466	.01304	.722
.96883	.421	.04582	1.499	.014[illegible]	.798
.96970	.439			.01497	.815

TABLE IV. (Continued)

RS Canum Venaticorum

Primary Minimum

Epoch-Phase	Luminosity	Epoch-Phase	Luminosity	Epoch-Phase	Luminosity
1978.01599	.869	1968.45690	1.486	1969.49787	1.422
.01679	.902	.45781	1.477	.49951	1.420
.01796	.940	.45855	1.473	.50048	1.418
.01889	.964	.45948	1.470	.50150	1.434
.02012	.993	.46022	1.455	.50268	1.444
.02087	1.003	.46110	1.467	.50341	1.455
.02193	1.019	.46187	1.460	.50431	1.447
.02268	1.113	.46278	1.477	.50598	1.454
.02371	1.159	.46360	1.446	.50679	1.436
.02447	1.206:	.46443	1.462	.50774	1.460
.02530	1.256:	.46525	1.462	.50848	1.444
.02610	1.186			.51207	1.466
.02686	1.259	1969.44828	1.441	.51291	1.469
.02845	1.326	.44918	1.488	.51394	1.445
.02919	1.343	.45015	1.475	.51476	1.469
.03029	1.392	.45101	1.575	.51578	1.460
.03107	1.418:	.45213	1.537	.51650	1.498:
.03288	1.434	.45294	1.496	.51748	1.459
.03372	1.425:	.45399	1.494	.51821	1.468
.0364	1.506:	.45474	1.499		
.03746	1.500	.45588	1.495	1971.53143.	1.540
.03864	1.549:	.45669	1.464	.53213	1.535
.03960	1.526	.45785	1.476	.53350	1.501
.04038	1.507	.45944	1.508	.53428	1.544
.04173	1.536	.46004	1.446:	.53545	1.526
.04278	1.523	.46106	1.466	.53619	1.548
.04357	1.523	.46198	1.488	.53706	1.565
.04468	1.537	.46441	1.482:	.53780	1.511
		.46675	1.438	.53870	1.479
Secondary Minimum		.46768	1.447		
		.46845	1.460:	1973.43753	1.564
1968.44056	1.499	.47001	1.452:	.43846	1.463
.44175	1.514	.48005	1.419	.43985	1.556
.44584	1.520:	.48085	1.414	.44201	1.484
.44678	1.514	.48181	1.412:	.44303	1.438
.44746	1.511	.48337	1.432	.44416	1.504
.44853	1.522	.48427	1.416	.44623	1.510
.44930	1.516	.48511	1.421	.44722	1.548
.45020	1.506	.48606	1.402	.44819	1.543
.45104	1.494	.48689	1.414	.45011	1.483
.45195	1.517	.49127	1.414:	.45124	1.480
.45269	1.501	.49223	1.425		
.45359	1.490	.49298	1.438	1974.46534	1.514
.45434	1.500	.49412	1.412	.46933	1.417:
.45528	1.479	.49548	1.398	.47113	1.478
.45621	1.484	.49634	1.407	.47298	1.471

TABLE IV. (Continued)

RS Canum Venaticorum

Secondary Minimum

Epoch-Phase	Luminosity
1974.47521	1.499
.47783	1.492
1975.49958	1.470
.50059	1.514
.50189	1.477
.51227	1.501:
.51308	1.498
.51404	1.458
.51501	1.442
.51569	1.439
.51696	1.485
.51789	1.485
.51913	1.479
.52026	1.440
.52158	1.419
.52237	1.512
.52331	1.484
1978.43424	1.477
.43536	1.523
.43639	1.594
.43761	1.504
.43842	1.546
.43950	1.551
.44062	1.467
.44160	1.539
.44285	1.560
.44389	1.539
.44486	1.511:
.45536	1.485:
.45664	1.490
.45773	1.488
.45859	1.473
.45959	1.508
1980.49730	1.430:
.49851	1.409
.50101	1.417
.50253	1.463:
.50440	1.424
.50715	1.400:
.50806	1.449:
.50935	1.441
.51080	1.488
.51252	1.509

Epoch-Phase	Luminosity
1980.51398	1.462
.51524	1.489
.51738	1.524
.51947	1.511
.52148	1.508
.52378	1.542
.52536	1.572
.52733	1.461
.53051	1.613
.53160	1.577
.53293	1.532
.53436	1.586
.53569	1.589
.53717	1.495

TABLE V.

RS Canum Venaticorum

Normal Points

Phase	Luminosity	Probable Error
.05779	1.535	.006
.07797	1.544	.006
.09575	1.551	.005
.10495	1.543	.005
.11423	1.546	.005
.12570	1.553	.004
.13849	1.570	.007
.15294	1.573	.006
.16309	1.602	.011
.17206	1.580	.004
.17866	1.578	.003
.18465	1.609	.005
.19019	1.603	.006
.20277	1.598	.005
.21164	1.590	.005
.21668	1.589	.004
.22190	1.596	.008
.22584	1.623	.005
.23212	1.595	.005
.23817	1.608	.005
.24509	1.597	.008
.24999	1.605	.004
.25629	1.596	.006
.26177	1.622	.009
.27314	1.619	.006
.30071	1.616	.005
.32514	1.608	.004
.34469	1.649	.006
.35787	1.601	.003
.36872	1.596	.002
.37623	1.625	.005
.38182	1.628	.006
.38865	1.596	.006
.39492	1.613	.012
.41107	1.549	.004
.41993	1.548	.005
.42496	1.542	.005
.43059	1.545	.006
.54403	1.505	.005
.55492	1.522	.008
.56577	1.501	.005
.57505	1.526	.005
.58665	1.536	.005
.59636	1.531	.004
.60246	1.535	.006
.61794	1.526	.002
.62949	1.534	.004
.63734	1.553	.003
.64403	1.537	.007
.65233	1.538	.005
.65856	1.547	.004
.66723	1.550	.002
.67945	1.557	.003
.69977	1.555	.003
.70856	1.561	.004
.71745	1.569	.005
.73388	1.569	.007
.75405	1.588	.006
.76011	1.568	.008
.76470	1.567	.005
.76983	1.564	.008
.77859	1.559	.007
.78668	1.539	.002
.79519	1.545	.003
.80362	1.536	.006
.81129	1.537	.004
.82647	1.532	.005
.83612	1.519	.003
.84313	1.524	.006
.84732	1.528	.008
.85210	1.529	.005
.86469	1.522	.004
.88014	1.519	.004
.89129	1.500	.009
.91046	1.508	.006
.91766	1.477	.004

TABLE VI.

RS Canum Venaticorum

Epoch and Phases of Observed Minima

Sitterly, From Harvard Plates

JD	Epoch + 3000 and Phase
12270	643.036
12679	728.049
15552	1327.031
15778	1374.028
15787	1376.026
15879	1395.032
16181	1458.026
16622	1550.028
16901	1608.021
16977	1624.025
17208	1672.018
17285	1688.016
17995	1836.020
18057	1849.019
18489	1939.015
19837	2220.007
19938	2241.010
20144	2284.005
20192	2294.007
20245	2305.008
20269	2310.008
20902	2442.018
20960	2454.001
20964	2455.006
21574	2582.003
21948	2659.999
22020	2674.999
23071	2893.995
23090	2898.003

Hoffmeister

JD	Epoch + 3000 and Phase
20298	2315.996
20302	2316.995
20346	2326.003
20370	2331.002
20423	2341.992
20489	2356.004
20566	2372.007
20653	2389.993
20681	2369.021
21267	2518.002
21607	2588.004
21982	2666.998
21986	2667.998

Sitterly, Visual Obs.

JD	Epoch + 3000 and Phase
22792	2835.998
22807	2838.998
22816	2840.999
22831	2843.998
22840	2845.999
23095	2898.997
23100	2099.999
23162	2912.999
23167	2913.997

Nijland & Godomski

JD	Epoch + 3000 and Phase
23579	3000.001

Schneller

JD	Epoch + 3000 and Phase
25249	3347.000

Himpel

JD	Epoch + 3000 and Phase
27873.524	3895.004

Keller & Limber

JD	Epoch + 3000 and Phase
33016.81924	4966.9832 ±.0003

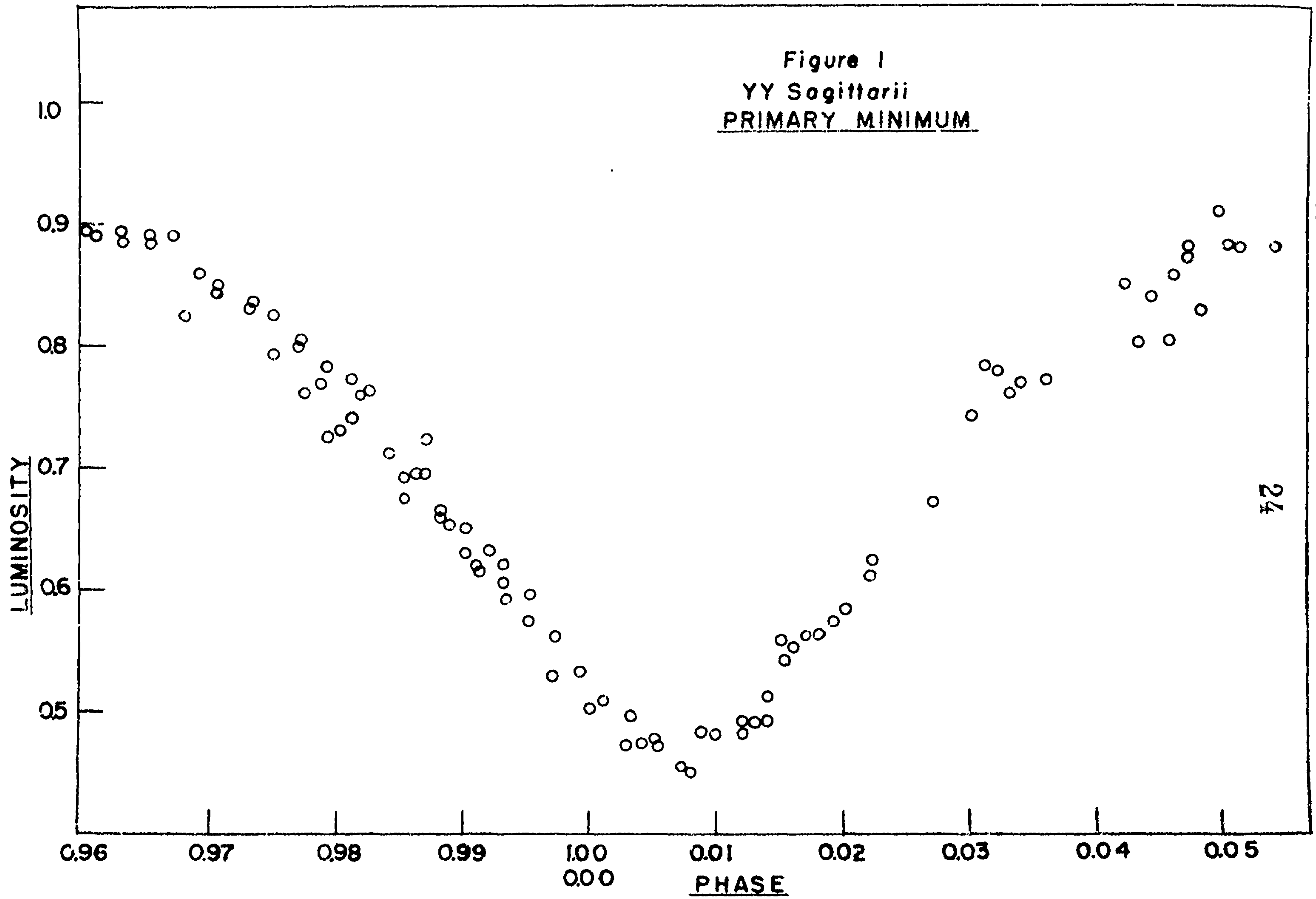
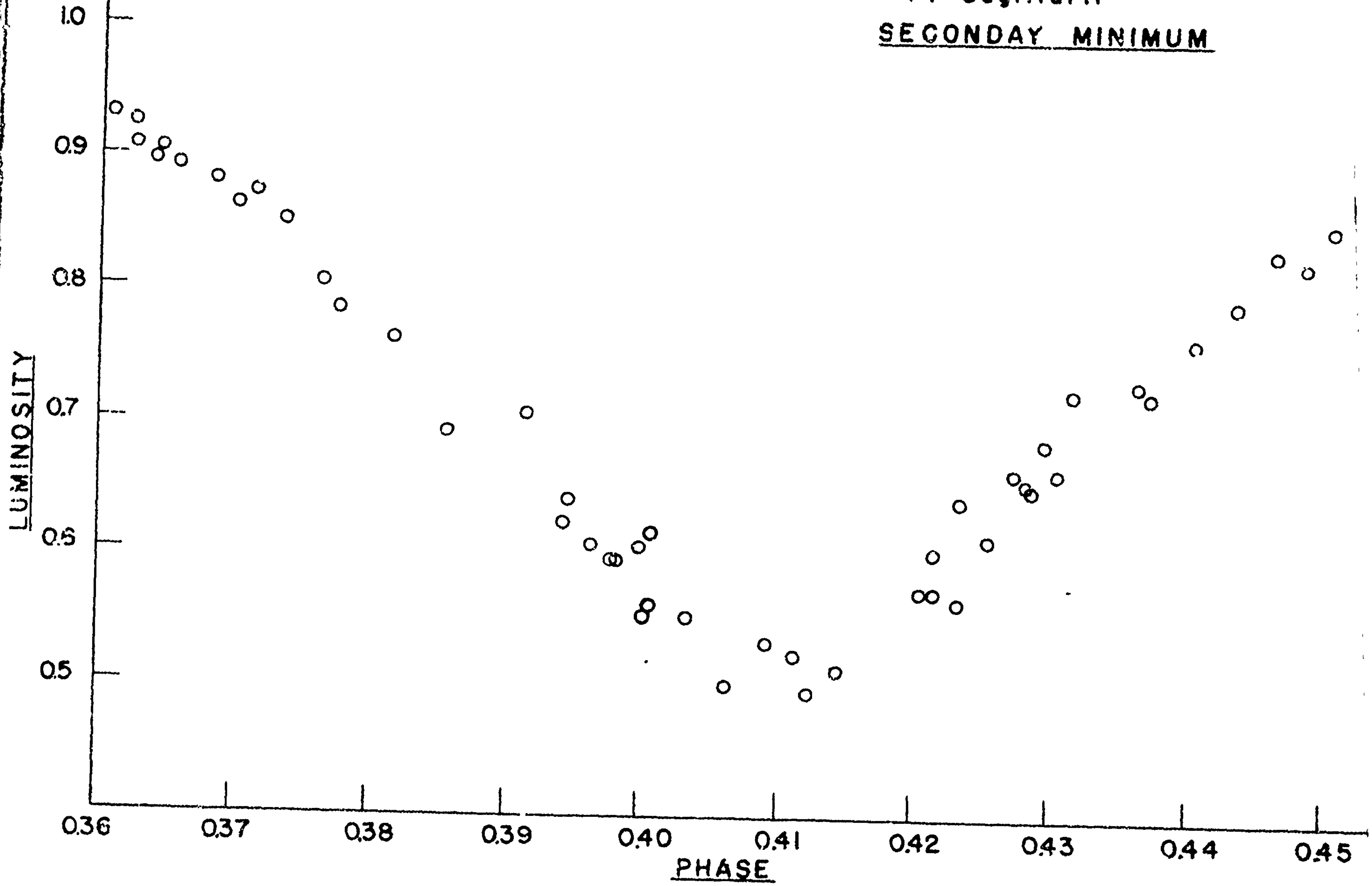


Figure I
YY Sagittarii
PRIMARY MINIMUM

Figure 2
YY Sagittarii
SECONDAY MINIMUM



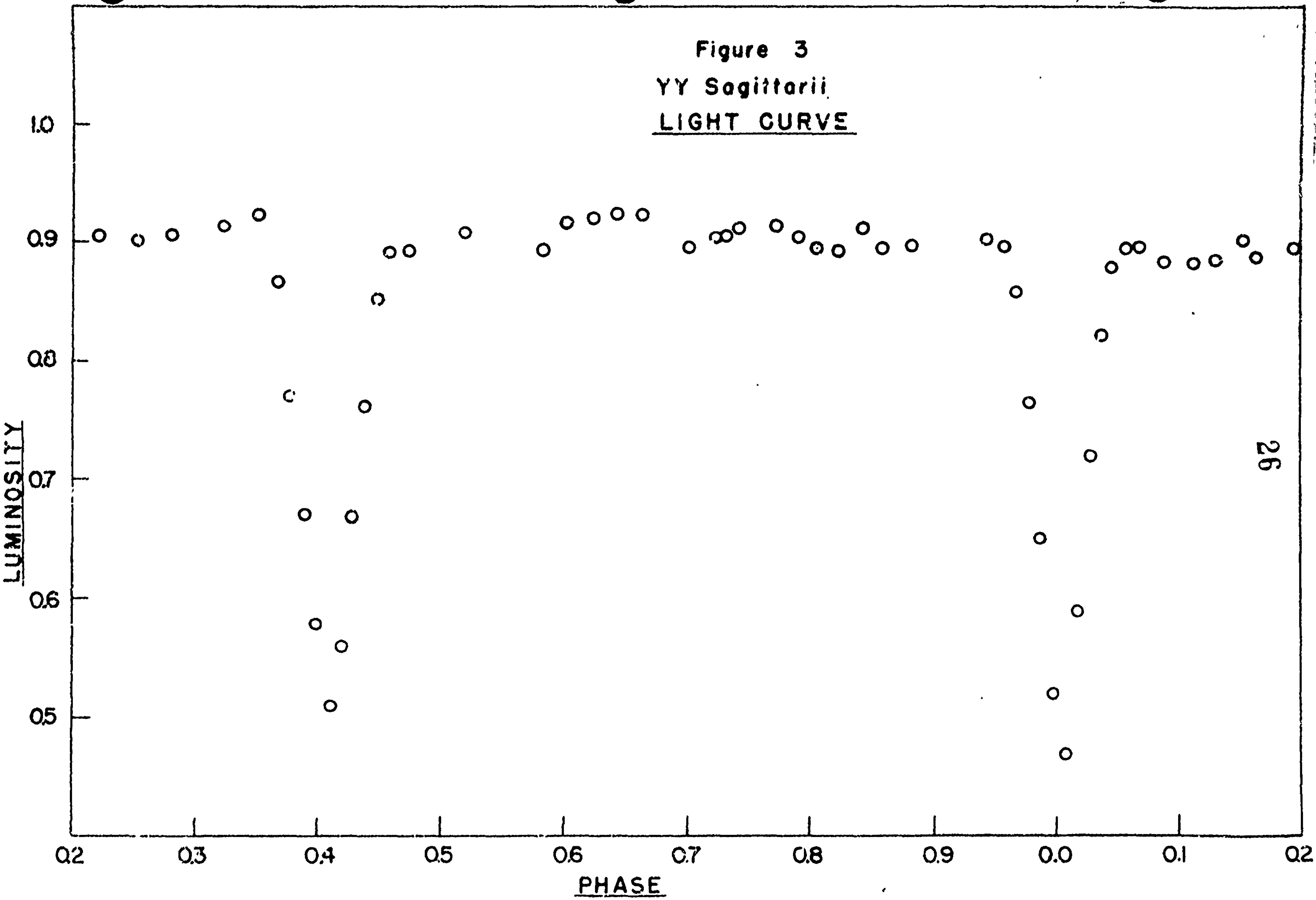


Figure 3
YY Sagittarii
LIGHT CURVE
LUMINOSITY
1.0
0.9
0.8
0.7
0.6
0.5
PHASE
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0.0
0.1
0.2
26

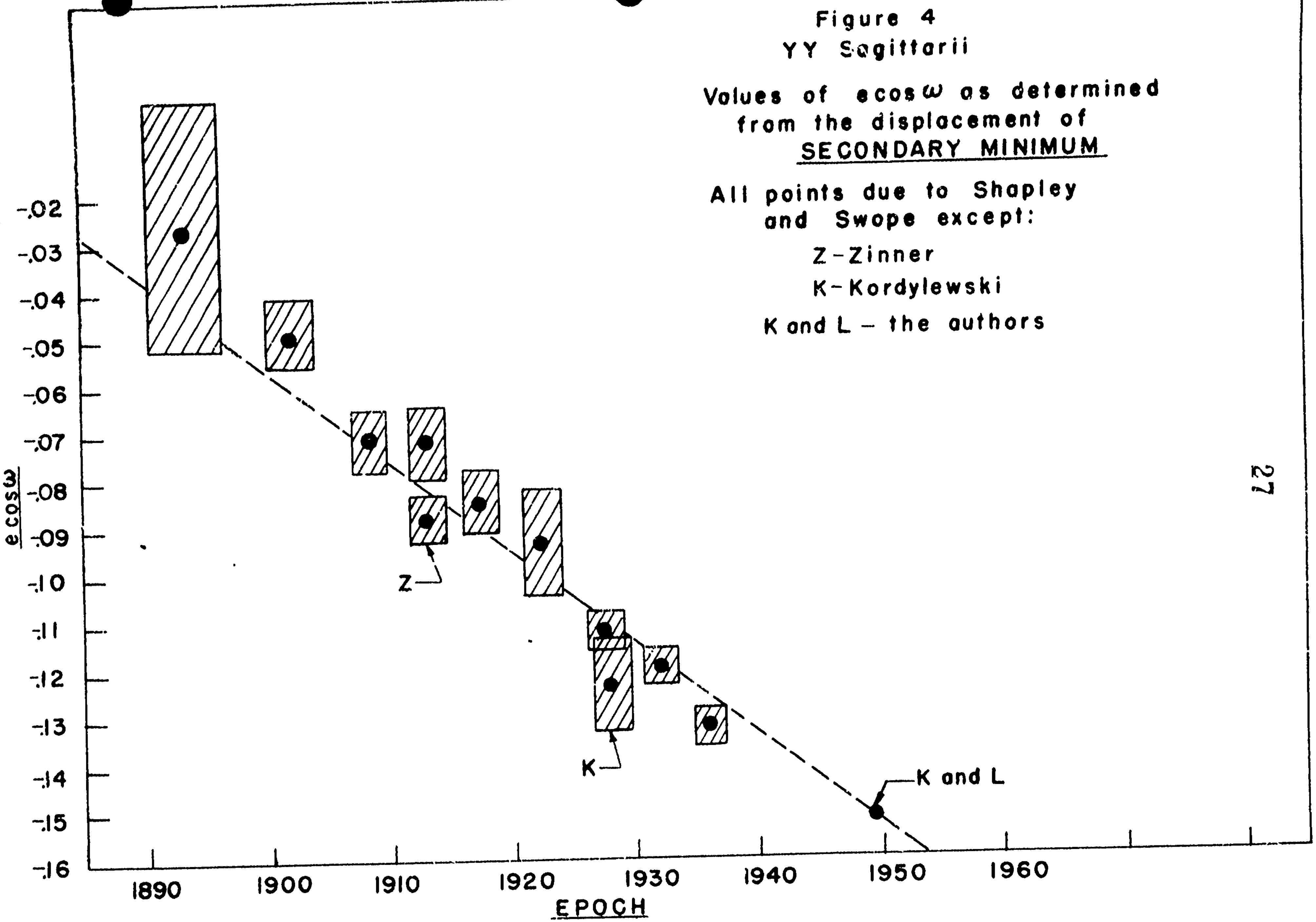


Figure 4
YY Sagittarii

Values of $e\cos\omega$ as determined from the displacement of SECONDARY MINIMUM

All points due to Shapley and Swope except:

Z – Zinner

K – Kordylewski

K and L – the authors

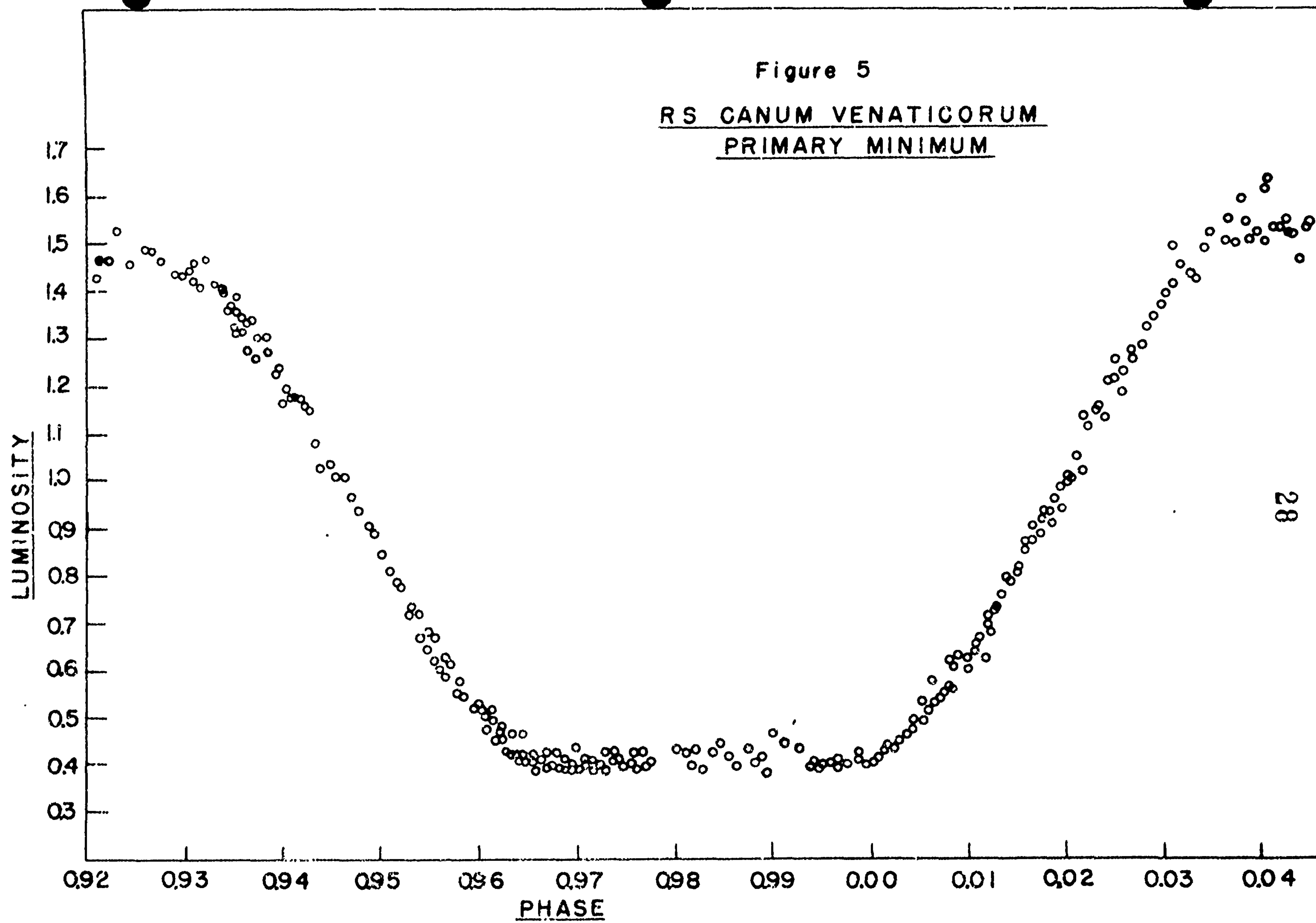


Figure 5

RS CANUM VENATICORUM
PRIMARY MINIMUM

Figure 6

RS CANUM VENATICORUM
SECONDARY MINIMUM

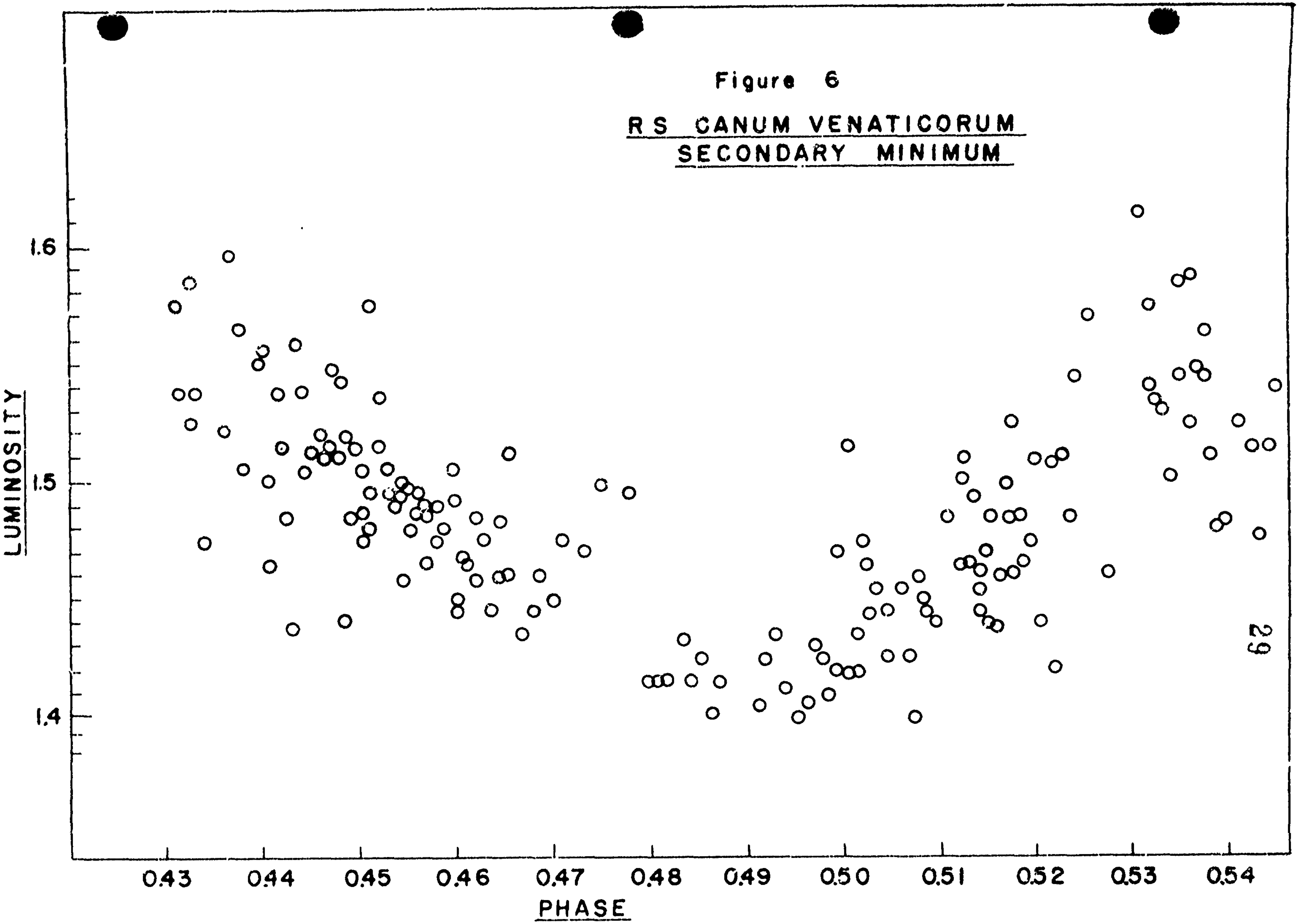
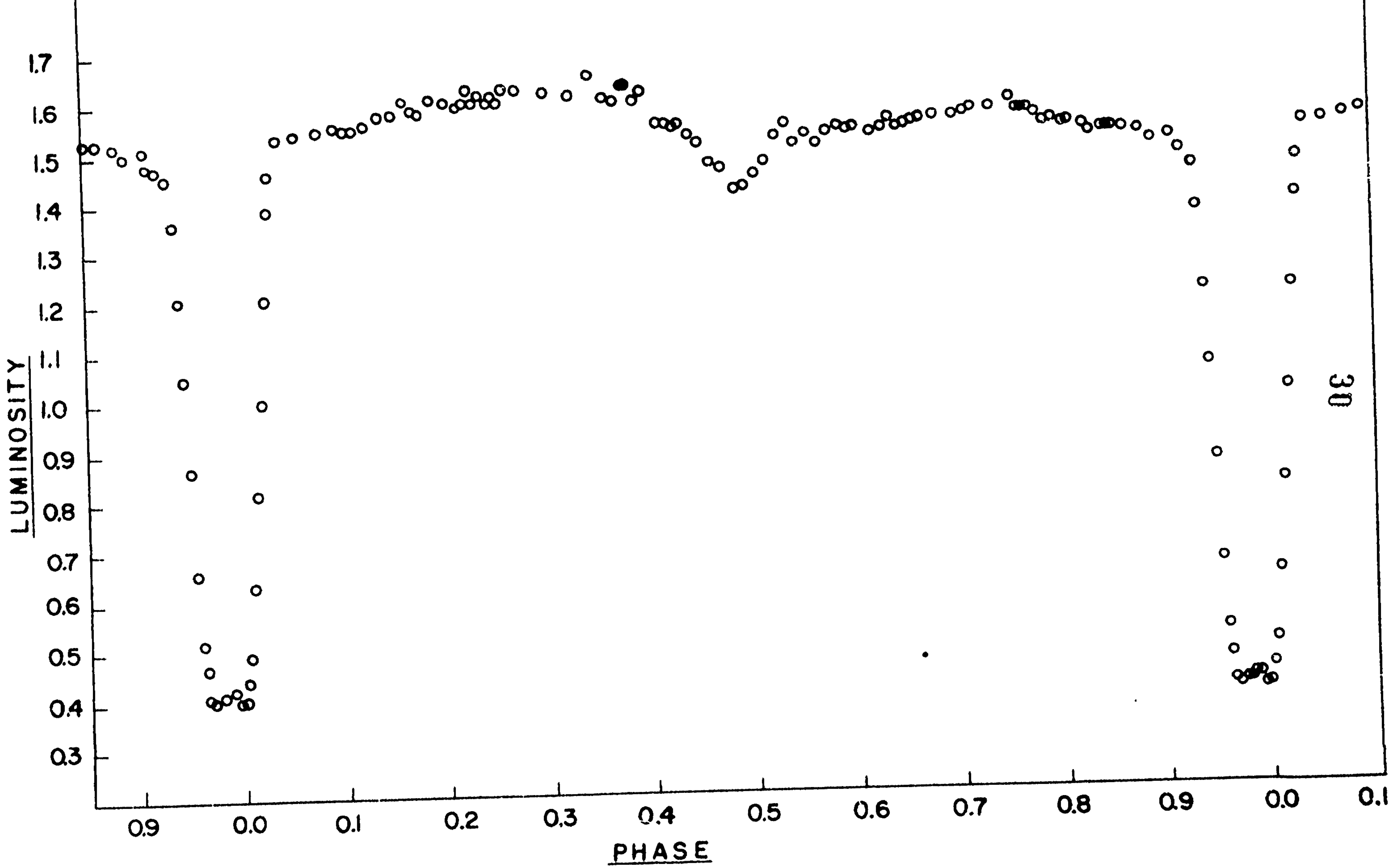


Figure 7

RS CANUM VENATICORUM
LIGHT CURVE



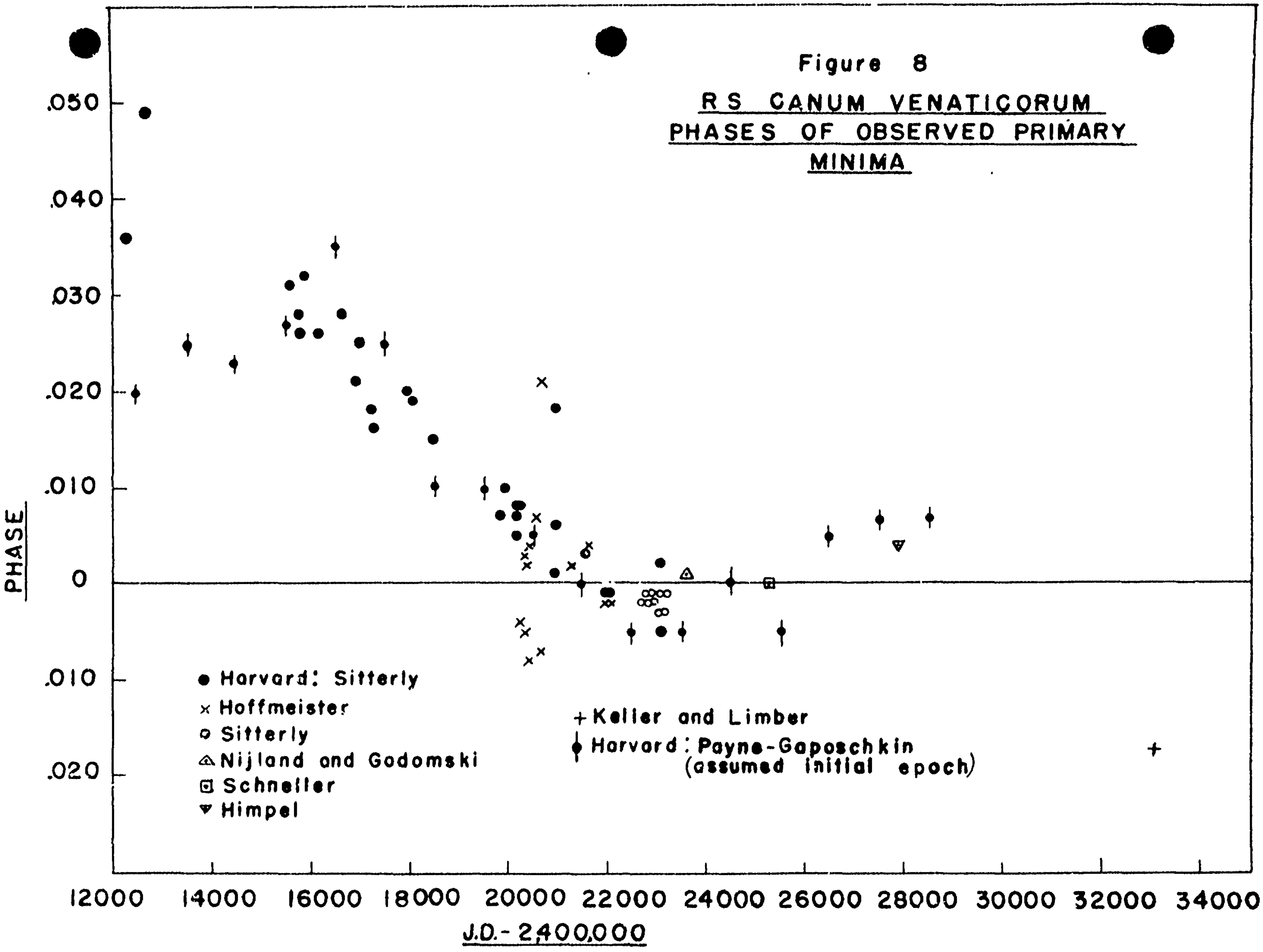


Figure 8

RS CANUM VENATICORUM
PHASES OF OBSERVED PRIMARY
MINIMA